PROCEEDINGS

OF

THE ROYAL SOCIETY.

December 5, 1889.

Sir G. GABRIEL STOKES, Bart., President, in the Chair.

The President announced that he had appointed as Vice-Presidents—

The Treasurer.
Professor Alfred Newton.
Sir Henry E. Roscoe.
Professor A. W. Williamson.

Professor T. McKenny Hughes was admitted into the Society.

Pursuant to notice, Professors Stanislao Cannizzaro, Auguste Chauveau, and Henry A. Rowland were balloted for and elected Foreign Members of the Society.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read :—

I. "Remarks on Mr. A. W. Ward's Paper 'On the Magnetic Rotation of the Plane of Polarisation of Light in doubly refracting Bodies.'" By O. WIENER and W. WEDDING, Physikalisches Institut, Strassburg i. E. Communicated by LORD RAYLEIGH, Sec. R.S. Received October 24, 1889.

In the above-mentioned paper Mr. Ward* communicates theoretical and experimental investigations which, as far as they are correct, are in their essential parts already published, and indeed somewhat more completely in three papers by Gouy† and ourselves,‡ the latter inves-

* Ward, 'Proceedings of the Royal Society,' 1889, vol. 46, p. 65.

† Gouy, 'Journal de Physique,' vol. 4, 1885, p. 149, "Sur les Effets simultanés du Pouvoir rotatoire et de la double Réfraction."

‡ Wiener, 'Wiedemann's Annalen,' vol. 35, 1888, p. 1, "Gemeinsame Wirkung von Circularpolarisation und Doppelbrechung, geometrisch dargestellt."

Wedding, *ibid.*, p. 25, "Die magnetische Drehung der Polarisationsebene bei wachsender Doppelbrechung in dilatirtem Glas."

tigations having been suggested by Professor Kundt. Experiments on the rotation of the plane of polarisation of light by means of interrupted currents, like those of Bichat and Blondlot, are here excluded. Since the above-mentioned papers have not been noticed by Mr. Ward, and are hence probably not well known, it may be of interest to reproduce here their essential contents.

Gouy shows that in a body, which at the same time has the power of double refraction and of rotation, certain vibrations, to which he gives the name "privilégiées," are propagated unaltered. These vibrations play exactly the same part in such bodies as the linear components in, and normal to, the principal section in the case of ordinary doubly refracting bodies, and as Fresnel's two circular components in the case of ordinary rotating bodies. In bodies which at the same time have the power of double refraction and of rotation, these two privileged components of vibration take place in opposite directions in two ellipses, whose major axes are one in the plane of principal section and the other perpendicular to it. The ratio of the axes and the difference of phase of the components are calculated by Gouy from the constants of double refraction and rotation.

For the complete solution of the problem of the propagation of light in such a body we only require to resolve any incident vibration into its two privileged components, and to compound them to a single resultant on emergence. The solution of this problem is contained in the above paper by Wiener, which was intended to supply the theoretical foundation for Wedding's investigation. It was especially important to determine how the rotation, due to the rotational power of the body, is disturbed by double refraction. It was found that the rotation alters periodically with the thickness of the plate, and with strong double refraction may even become negative, that is to say, in the opposite direction to that due to the rotational power. The rotation is zero in the neighbourhood of those places where the difference of phase of the linear components, due to double refraction, is a multiple of π, and not, as Mr. Ward thinks, of $\pi/2$. The general result of Wiener's paper may be quoted as follows:—

"Herrscht bei der gleichzeitigen Wirkung von Circularpolarisation* und Doppelbrechung die eine vor, so wird die andere theilweise verdeckt. Eine starke Doppelbrechung drückt die Drehung der Circularpolarisation und eine starke Circularpolarisation das Elliptischmachen der Doppelbrechung herab."

Wedding's investigation is of purely experimental nature, and consists of two parts. The first is occupied with magnetic rotation in stressed glass, and completely confirms by experiment the con-

* By "Circularpolarisation" is understood what we have here called rotational power.

clusions of the theory, especially with regard to zero and negative rotation. It concludes with the words:

"Ebenso darf man bei Krystallen, an denen man keine elektromagnetische Drehung beobachten kann, nicht den Schluss ziehen, dass ihnen keine Verdet'sche Constante zukäme, weil eine vorhandene Drehung durch die superponirte Doppelbrechung geschwächt wird und bis zur Unmerklichkeit verdeckt werden kann."

In the second part he communicates the repetition of Villari's experiment on the rotation of the plane of polarisation of light in a disk spinning between magnet poles. He finds a diminution of rotation from 5·06° to 0·77° when the disk spins 10,800 times in a minute, and proves experimentally that this is due to the double refraction produced by centrifugal force. For, on the one hand, the double refraction produces a difference of phase in the linear components, which, like the centrifugal force, is proportional to the square of the number of revolutions, and hence must be due to this force; on the other hand, the rotation vanishes as the theory requires, just where the difference of phase of the linear components is π.

Thus Mr. Ward's essential results, namely, the diminution of the rotation by double refraction, and the explanation of Villari's experiment by means of the double refraction due to centrifugal force, have been already communicated in the above-named papers.

With regard to Mr. Ward's mathematical deductions, the first of the before-mentioned authors wishes to point out a mistake which Mr. Ward has made in forming his differential equation.

In accordance with Mr. Ward, we let the x or y axis fall in the plane of principal section, and the z axis in the direction of the rays of light. We must examine how an ellipse is altered under the common action of rotation and double refraction, if we advance in the medium a short distance dz. Like Mr. Ward, we take the ellipse as produced by composition of y and x components, the ratio of whose amplitudes is $\tan \alpha$, and whose difference of phase is β; then the direction of the major axis forms with the x axis an angle ω, determined by the equation

$$\tan 2\omega = \tan 2\alpha \cos \beta. \qquad (1.)$$

Since the double refraction does not alter the value of α, we obtain the variation of ω with β under the influence of double refraction alone by forming $\partial\omega/\partial\beta$, regarding α as constant—

$$\frac{\partial\omega}{\partial\beta} = -\tfrac{1}{2}\cos^2 2\omega \tan 2\alpha \sin \beta,$$

or

$$\frac{\partial\omega}{\partial\beta} = -\tfrac{1}{4}\sin 4\omega \tan \beta. \qquad (2.)$$

So far our calculation agrees with Mr. Ward's.

To continue: let β vary under the influence of *double refraction* alone by $\partial\beta_\delta = k\,\partial z$, when the vibration advances through ∂z; then the alteration of ω is $\partial\omega = -\frac{1}{4}k \sin 4\omega \tan\beta\,\partial z$.

Under the influence of *rotation* alone, let ω vary by $\partial\omega_r = m\,\partial z$; then the total alteration of ω with z is given by the equation—

$$\frac{d\omega}{dz} = -\frac{k}{4}\sin 4\omega \tan\beta + m. \quad \ldots\ldots\ldots\ldots\ldots \quad (3.)$$

In this equation β is still unknown; it is also variable with the double refraction and with the rotation. We must therefore form a second differential equation for β. We already know the alteration of β due to *double refraction* alone; it is $\partial\beta_\delta = k\,\partial z$. In order to learn the variation of β with *rotation* alone, we are in need of a relation between β and ω, in which only β and ω are variable in consequence of rotation. This relation is—

$$\tan\beta \sin 2\omega = \mathrm{K}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.)$$

where

$$\mathrm{K} = -\frac{2k}{1-k^2},$$

if k is the ratio of the minor and major axes of the ellipse. This ratio, as a matter of fact, does not alter with rotation.

From this follows—

$$\frac{\partial\beta}{\partial\omega} = -\frac{\sin 2\beta}{\tan 2\omega}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.)$$

But since $\partial\omega_r = m\,\partial z$, the variation of β by rotation alone is $\partial\beta_r = -m\dfrac{\sin 2\beta}{\tan 2\omega}\,\partial z$. The total variation of β with z is hence given by the equation—

$$\frac{d\beta}{dz} = k - m\frac{\sin 2\beta}{\tan 2\omega}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.)$$

Thus the complete solution of the problem is contained in the two simultaneous differential equations—

$$\left.\begin{aligned} \frac{d\omega}{dz} &= -\frac{k}{4}\sin 4\omega \tan\beta + m \\ \frac{d\beta}{dz} &= k - m\frac{\sin 2\beta}{\tan 2\omega}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (7.)$$

Mr. Ward's solution, however, consists in the single differential equation—

$$\frac{d\omega}{dz} = -\frac{k}{4}\sin 4\omega \tan kz + m. \quad \ldots\ldots\ldots\ldots\ldots \quad (8.)$$

He obtains it by setting in equations (2) and (3) $\beta = k.z$. This relation is not correct, since it only represents the variation of β with double refraction, whilst β also varies with rotation.

To show the effect of this mistake, we will consider the particularly simple case where the incident vibration is along the x axis alone, and where the double refraction is so much stronger than the rotation that we can replace $\sin \omega$ by ω.

The correct differential equations are—

$$\left.\begin{aligned} \frac{d\omega}{dz} &= -k\omega \tan \beta + m \\ \frac{d\beta}{dz} &= k - \frac{m}{2\omega}\sin 2\beta. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (9.)$$

Mr. Ward's is—

$$\frac{d\omega}{dz} = -k\omega \tan kz + m. \quad \ldots\ldots\ldots\ldots\ldots \quad (10.)$$

The solution of the simultaneous differential equations (9) is—

$$\left.\begin{aligned} \omega &= \frac{m}{k}\sin kz \\ \beta &= \frac{k}{2}z, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11.)$$

whilst Mr. Ward obtains for this case by integration of (10)—

$$\omega = \frac{m}{k}\cos kz \log_e \tan\left(\frac{\pi}{4}+\frac{kz}{2}\right). \quad \ldots\ldots\ldots\ldots \quad (12.)$$

From (11) it follows in accordance with experiment that $\omega = 0$ when kz, the difference of phase, is a multiple of π. Mr. Ward concludes from (12) that $\omega = 0$ when kz is a multiple of $\pi/2$.

To conclude: Mr. Ward's single differential equation must be replaced by two simultaneous differential equations. The conclusions which Mr. Ward draws from his equation are hence in part incorrect, in part not properly proved.

[Reference may also be made to Professor Willard Gibbs's investigation of "Double Refraction in perfectly Transparent Media which exhibit the Phenomena of Circular Polarisation," 'Amer. Journ. Sci.,' June, 1882.—R.]